

Susceptibility amplitude ratio for generic competing systems

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Abstract

We calculate the susceptibility amplitude ratio near a generic higher character Lifshitz point up to one-loop order. We employ a renormalization group treatment with L independent scaling transformations associated to the various inequivalent subspaces in the anisotropic case in order to compute the ratio above and below the critical temperature and demonstrate its universality. Furthermore, the isotropic results with only one type of competition axes have also been shown to be universal. We describe how the simpler situations of m -axial Lifshitz points as well as ordinary (noncompeting) systems can be retrieved from the present framework.

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I. INTRODUCTION

The occurrence of m -axial Lifshitz phase transitions [1, 2] in various real physical systems (e.g., magnetic modulated materials [3–8], high- T_c superconductors [9–11], liquid crystals [12–14], etc.) has increased the interest in the field-theoretic description of this subject in the last few years [15–17]. According to modern renormalization group arguments, critical phenomena of m -axial Lifshitz competing systems have their universality classes characterized by (N, d, m) , namely, the number of components of the (field) order parameter N dwelling in d space dimensions with $m(\equiv m_2)$ space directions presenting alternate (repulsion-attraction) among the fields [18–20]. Another type of “competing axes” can be defined: if the alternate couplings are of the type attractive-repulsive-attractive and take place along m_3 spatial dimensions, one refers to m_3 -axial third-character Lifshitz critical behavior (for the realization of the $m_3 = 1$ case, see [21]).

More generally, a m_L -fold L -th character Lifshitz behavior appears whenever short-ranged alternate interactions with L couplings of the type repulsion-attraction-repulsion-attraction-... are allowed [22–24]. If all sorts of the aforementioned competing axes are present simultaneously in the critical system under consideration, its phase transitions are governed by the generic higher character Lifshitz critical behavior. The universality classes of these arbitrary competing systems are defined by the set (N, d, m_2, \dots, m_L) [25–27]. The language of magnetic systems is particularly suitable to describe these systems. It is convenient to make the connection of these complex critical behaviors with the prototype of second order phase transitions in noncompeting systems: the Ising model ($N = 1$) [28].

The simplest realization of the usual Lifshitz universality class (N, d, m) can be encountered in uniaxial critical (systems $m = 1$) behavior. It can be understood in terms of the axial next-nearest-neighbor Ising (ANNNI) model [29, 30] which corresponds to the usual Ising model including antiferromagnetic exchange interactions among second neighbor spins along a single axis in a cubic lattice. The uniaxial Lifshitz point arises at the confluence of the disordered, a uniformly ordered and a modulated phase. Anisotropic second character m -axial points generalize that uniaxial when the competing axes occur along m space directions whenever $m \neq d$.

The ANNNI model can be generalized by including further ferromagnetic couplings among third neighbors along the competing axis. Except for a little additional complication in the

phase diagrams due to the existence of an additional parameter related with the third neighbor coupling, similar phases can be defined such that the region of intersection of them terminates in a point where the different phases characterizing the system meet, known as the uniaxial Lifshitz point of third character [21]. This reasoning can be extended to contemplate the situation where alternate couplings up to the L -th neighbors exist, and the critical point associated to the region of confluence of the several phases of the system is denominated the uniaxial Lifshitz point of L -th character. If the system presents competing interactions of this type along $m_L \neq d$ ($= d$) space directions, the system is said to have anisotropic (isotropic) critical behavior with m_L -axial point of L -th character [22–24]. The modulated phases have a distinction when we compare anisotropic and isotropic behaviors. In the former there exist two types of correlation lengths, namely ξ_1 and ξ_L , which label two inequivalent subspaces characterized by correlations perpendicular and parallel, respectively, to the m_L subspace. In isotropic behaviors only one type of correlation length ξ_L characterizes each modulated phase.

The anisotropic Lifshitz point of generic L th character can also be defined in the description of the most general d -dimensional competing system, whenever several types of competing axes show up *simultaneously*. Let us consider its simplest realization. In that case, there are only nearest neighbor interactions along $m_1 \equiv (d - m_2 - \dots - m_L)$ (noncompeting) directions, second neighbor competing interactions along m_2 directions perpendicular to the m_1 dimensions, competition among third neighbors along m_3 space directions (orthogonal to the (m_1, m_2) subspaces), etc., up to L th neighbor alternate couplings along m_L directions, with all competition subspaces perpendicular to each other. The model which describes this sort of arbitrary competing systems was proposed a few years ago and named *competing exchange coupling Ising (CECI)* model [25]. There are L inequivalent correlation lengths owing to the L independent competing axes m_n ($n = 1, \dots, L$). This situation allows in principle low temperature $(L - 1)$ modulated phases in equilibrium with the uniform ordered phase as well as with the high temperature disordered phase close to the Lifshitz point. In addition, it is also possible that the complex systems can display several low temperature (up to $L - 1$) uniformly ordered phases in equilibrium with (at least one) modulated ordered and high temperature disordered phases.

The isotropic m -axial critical behavior has been experimentally realized in the context of polymers. At first the isotropic behavior was thought of being of purely academic interest.

Nevertheless, its theoretical mean-field prediction in copolymer-homopolymer ternary blends [31] and subsequent experimental identification in mixtures of block copolymer-homopolymer [32] caused a certain enthusiasm, but the following paper with a more detailed analysis on the subject showed a microemulsion phase incompatible with the existence of the Lifshitz point [33]. It was argued there that the fluctuations destroyed this multicritical point, although the associated critical region could be identified with the vicinity of the would be mean field Lifshitz point. The theoretical effect of fluctuations was incorporated immediately afterward using a self-consistent field theory (SCFT), and the ordered lamellar phase previously identified as the modulated phase was understood utilizing a one-component order parameter ($N = 1$) [34]. This result not only confirmed the previous discussion from Ref.[33], but also located loci in the mean field phase diagram with third character isotropic Lifshitz point. Later, mean field studies using SCFT indicated the existence of up to 6th character isotropic critical behavior in blends of diblock copolymers [35]. This suggests that the isotropic behaviors of the *CECI* model might be useful in unveiling properties of these real physical systems, even though the anisotropic realization of this model has not been identified yet.

Thence, several experiments have been performed for these polymers. For instance, the susceptibility of a homopolymer-diblock copolymer blend (polybutadiene and polystyrene) has been investigated recently using small angle neutron scattering. Some amplitudes above the Lifshitz temperature were estimated for fixed values of the diblock copolymer composition [36]. The closest we can get to this system using field theory techniques is to look for universal quantities related to the susceptibility, i.e., the amplitude ratio above and below the Lifshitz critical temperature [37]. By the same token, the study of a simple property such as the susceptibility for the *CECI* model could shed light on possible future experiments related with real physical systems manifesting this especial critical behavior.

In this paper, the susceptibility amplitude ratio for generic competing systems will be computed using field theory and renormalization group arguments up to one-loop order in a perturbation expansion. The anisotropic behaviors with arbitrary types of competing axes are discussed first. We are going to restrict ourselves to (fields) order parameters of only one component ($N = 1$). The results are presented in a manifestly universal form and are shown how to reduce to the ordinary amplitude ratio without competition. We then restrict the number of competition axes to obtaining information on particular universality classes, the

most obvious being the m -axial anisotropic Lifshitz criticalities. We show that the uniaxial result can be retrieved from the arbitrary anisotropic competing systems in a simple manner. The isotropic amplitude ratios for isotropic critical behaviors are calculated for the first time. The m -axial universality class is recognized from the generic situation whenever $n = 2$. We show that although the expansion parameter is large for three-dimensional systems, our perturbative results are meaningful for those systems. As an application, we compare our field-theoretic $m = 3$ isotropic output with experimental results from homopolymer-diblock copolymer mixtures and show very good agreement among them.

The paper is organized as follows. In Section II we present the one-loop effective potential. We highlight a brief explanation of the several subspaces which occur in the problem along with the independent renormalization group transformations in the anisotropic cases. A simpler analogous discussion for calculating the isotropic amplitude ratio is explicated in Section III. Section IV presents the discussion of the results and conclusions.

II. ANISOTROPIC AMPLITUDE RATIO FOR GENERIC COMPETING SYSTEMS

We begin with the bare Lagrangian density associated with anisotropic generic competing systems described by the *CECI* model, which is given by

$$\begin{aligned}
L = & \frac{1}{2} \left| \nabla_{\left(d - \sum_{n=2}^L m_n\right)} \phi_0 \right|^2 + \sum_{n=2}^L \frac{\sigma_n}{2} \left| \nabla_{m_n}^n \phi_0 \right|^2 \\
& + \sum_{n=2}^L \delta_{0n} \frac{1}{2} \left| \nabla_{m_n} \phi_0 \right|^2 + \sum_{n=3}^{L-1} \sum_{n'=2}^{n-1} \frac{1}{2} \tau_{nn'} \left| \nabla_{m_n}^{n'} \phi_0 \right|^2 \\
& + \frac{1}{2} t_0 \phi_0^2 + \frac{1}{4!} \lambda_0 \phi_0^4.
\end{aligned} \tag{1}$$

The parameters which correspond to the physical situations are the coefficients of the derivatives of the bare field ϕ_0 (order parameter of the phase transition), the bare reduced temperature $t_0 (\propto T - T_L)$, where T_L is the Lifshitz critical temperature) and the bare coupling constant λ_0 .

The Lifshitz critical region is defined for particular combinations of the exchange interactions among all the neighbors. This implies the fine-tuning conditions on some parameters, namely, $\delta_{0n} = \tau_{nn'} = 0$.

At the Lifshitz critical region, the temperature is close but not equal to T_L . The structure of the field theory considered at this region is such that its momentum dependence on the various competing subspaces is rather peculiar. There are quadratic momenta components along the m_1 -dimensional noncompeting subspace, quartic momenta components along the m_2 -dimensional competing subspace, and so on, up to the $2L$ -th power of momenta along the m_L -dimensional subspace, which are present simultaneously in the free bare critical propagator in momentum space. We can set $\sigma_n = 1$ provided we perform a dimensional redefinition in the momentum characterizing the n -th m_n -dimensional competition subspace. If Λ is a momentum scale, we take the engineering dimension of the competing subspace as $[k_{(n)}] = \Lambda^{\frac{1}{n}}$.

The anisotropic behaviors is characterized by L independent correlation lengths ξ_n , one for each subspace. They induce L independent renormalization group flows in the parameter space of the massless theory. If we use normalization conditions in the definition of the renormalized theory, these flows can be described by L independent sets of normalization conditions, each of them defining a symmetry point SP_n ($n = 1, \dots, L$) which simplifies our task of computing universal quantities in this formalism of one-particle irreducible (1PI) vertex parts.

Typically, The Feynman integrals involved depend on various external momenta scales, namely that characterizing the $(d - m_2 - \dots - m_L)$ -dimensional noncompeting subspace, a momentum scale associated to the m_2 space directions, etc., up to the momentum scale corresponding to the m_L competing axes. For example, an explicit integral that shall be used is the one-loop contribution to the four-point function, namely

$$I(P, K'_{(2)}, \dots, K'_{(L)}) = \int \frac{d^{(d - \sum_{n=2}^L m_n)} q \prod_{n=2}^L d^{m_n} k_{(n)}}{[\sum_{n=2}^L ((k_{(n)} + K'_{(n)})^2)^n + (q + P)^2] \left(\sum_{n=2}^L (k_{(n)}^2)^n + q^2 \right)} . \quad (2)$$

Although this integral should be computed in arbitrary nonvanishing external momenta components, in practice the calculation is simplified when we choose only one subspace, say m_n , whose momenta are set in the arbitrary value $K'_{(n)}{}^{2n} = \kappa_n^{2n}$. In case we wish to determine universal quantities associated to vertex parts along the j th type of competing axes, we set $\kappa_n = 0$ for $n \neq j$ maintaining, however, $\kappa_j \neq 0$.

Unfortunately, the integral cannot be solved exactly for arbitrary external momenta, but

can be resolved using the orthogonal approximation which permits to obtain the integral as an homogeneous function of the external momenta. Within this framework, the Feynman integrals of the corresponding field theory can be computed to all loop orders. In normalization conditions, the results for those integrals are independent of the subspace chosen.

In order to be precise in our description, we should label the renormalized vertex parts according to the subspace characterized by the nonvanishing momenta scale κ_n associated to the symmetry point SP_n . Fortunately, we do not have to employ this label in the present work, since in the context of the orthogonal approximation all renormalization directions possess the same fixed point [25]. In other words, the susceptibility amplitude ratio is independent of the renormalization group transformation characterized by the variation of the external momenta scale κ_n in the renormalized vertex parts. In what follows, we shall use for simplicity all vertex parts computed at the symmetry point $SP_1 \equiv SP$, i.e., the external momenta scale is given by $\kappa_n = \delta_{1n}\kappa$ with $P^2 = \kappa^2 = 1$. With this choice we do not need to employ the orthogonal approximation to perform this integral in the anisotropic case, since the nonvanishing external momenta is contained in the quadratic term of the propagator. This quadratic part can be evaluated to arbitrary external momenta and the resulting expression for this integral is exact. For further details, the reader is advised to consult Ref. [26].

The bare quantities can be transformed into renormalized amounts at one-loop level through the renormalization of the bare field and temperature which are given by $t_0 = Z_{\phi^2}^{-1}t$, $\phi = Z_{\phi}^{-\frac{1}{2}}M$ beside the renormalized coupling constant. We express the latter in the fixed point in terms of the dimensionless entity u^* as $g^* = u^*\kappa^{\epsilon_L}$ where $\epsilon_L = 4 + \sum_{n=2}^L \frac{(n-1)}{n}m_n - d$ is the perturbation parameter.

Using the symmetry point, let us write down the one-loop renormalized Helmholtz free energy density at the fixed point. It is simply the renormalized effective potential at one-loop plus polynomial terms in t used to define additively renormalized vertex parts. For completeness (and anticipating future discussions for the specific heat amplitude ratio as well) we include a term proportional to t^2 which, however, will have no consequence to our discussion in the present work. Putting those arguments together, we obtain the following

expression

$$\begin{aligned}
F(t, M) = & \frac{1}{2}tM^2 + \frac{1}{4!}g^*M^4 + \frac{1}{4}(t^2 + g^*tM^2 + \frac{1}{4}(g^*M^2)^2)I_{SP} \\
& + \frac{1}{2} \int d^{(d-\sum_{n=2}^L m_n)} q \left[\prod_{n=2}^L d^{m_n} k_{(n)} \right] \left[\ln \left(1 + \frac{t + \frac{1}{2}g^*M^2}{\left(\sum_{n=2}^L (k_{(n)}^2)^n + q^2 \right)} \right) \right. \\
& \left. - \frac{g^*M^2}{2 \left(\sum_{n=2}^L (k_{(n)}^2)^n + q^2 \right)} \right], \tag{3}
\end{aligned}$$

where in the above equation t, M ($t_0 = Z_{\phi^2}^{-1}t, \phi = Z_{\phi}^{-\frac{1}{2}}M$) are the renormalized (bare) reduced temperature and order parameter, respectively, Z_{ϕ^2}, Z_{ϕ} are normalization functions, g^* is the renormalized coupling constant at the fixed point, \vec{q} is a $(d-m)$ -dimensional wave vector perpendicular to the competing axes, whereas \vec{k} is a m -dimensional wave vector whose components are parallel to the competition axes. The integral I_{SP} is defined by:

$$I_{SP} = \int \frac{d^{(d-\sum_{n=2}^L m_n)} q \prod_{n=2}^L d^{m_n} k_{(n)}}{\left[\sum_{n=2}^L k_{(n)}^{2n} + (q+P)^2 \right] \left(\sum_{n=2}^L k_{(n)}^{2n} + q^2 \right)}, \tag{4}$$

where the convenient symmetry point for this integral is defined as above, namely, $P^2 = \kappa^2 = 1$. This choice has the virtue of transforming the dimensionful coupling constant in its dimensionless version, i. e., $g^* = u^*$ and is the most effective route to computing universal quantities in the context of the renormalization group strategy. Whenever a loop integral is performed, a typical geometric angular factor is produced, which can be factored out in a redefinition of the coupling constant in a standard way [28]. In our case this factor is given by the expression $[S_{(d-\sum_{n=2}^L m_n)} \Gamma(2 - \sum_{n=2}^L \frac{m_n}{2n}) (\prod_{n=2}^L \frac{S_{m_n} \Gamma(\frac{m_n}{2n})}{2n})]$, such that it is going to be omitted whenever we report the result of any loop integral. The last integral at the symmetry point was already computed in Ref.[26] and shown to be given by $I_{SP} = \frac{1}{\epsilon_L}(1 + h_{m_L}\epsilon_L)$, where $h_{m_L} = 1 + \frac{(\psi(1) - \psi(2 - \sum_{n=2}^L \frac{m_n}{2n}))}{2}$ and $\psi(z) = \frac{d \ln \Gamma(z)}{dz}$. It is worthy to stress that whenever $m_3 = \dots = m_L = 0$, $h_{m_2} = [i_2]_m$ and the usual anisotropic m -axial Lifshitz critical behavior is obtained from this more general competing situation in a rather simple manner.

Since we need the value of M in the coexistence curve above and below T_L , let us compute the renormalized magnetic field, which is given by

$$H_R = \frac{\partial F}{\partial M} = tM + \frac{1}{6}u^*M^3 + \frac{u^*M}{2}(t + \frac{u^*M^2}{2})[I_{SP} - I], \tag{5}$$

where

$$I = \int \frac{d^{(d-\sum_{n=2}^L m_n)} q \prod_{n=2}^L d^{m_n} k_{(n)}}{[\sum_{n=2}^L ((k_{(n)})^2)^n + q^2] \left(\sum_{n=2}^L (k_{(n)}^2)^n + q^2 + t + \frac{u^* M^2}{2} \right)}. \quad (6)$$

Let us compute explicitly this integral. First we use a Feynman parameter to write it as

$$I = \int_0^1 dx \int \frac{d^{(d-\sum_{n=2}^L m_n)} q \prod_{n=2}^L d^{m_n} k_{(n)}}{[\sum_{n=2}^L ((k_{(n)})^2)^n + q^2 + x(t + \frac{u^* M^2}{2})]^2}. \quad (7)$$

In order to integrate the quadratic momentum out, we employ the identity

$$\int d^d q \frac{1}{[q^2 + 2kq + m^2]^\alpha} = \frac{S_d \Gamma(\frac{d}{2}) \Gamma(\alpha - \frac{d}{2})}{2\Gamma(\alpha)} (m^2 - k^2)^{\frac{d}{2} - \alpha}, \quad (8)$$

and get to

$$I = \frac{1}{2} S_{(d-\sum_{n=2}^L m_n)} \Gamma(d - \sum_{n=2}^L m_n) \Gamma(2 - (d - \sum_{n=2}^L m_n)) \int_0^1 dx \int \prod_{n=2}^L d^{m_n} k_{(n)} \\ \times \frac{1}{[\sum_{n=2}^L ((k_{(n)})^2)^n + x(t + \frac{u^* M^2}{2})]^{2 - \frac{(d-\sum_{n=2}^L m_n)}{2}}}. \quad (9)$$

Now we have to perform the remaining integral with higher power of momentum. Indeed, in the integral

$$i_n = \int \frac{d^{m_n} k_{(n)}}{[\sum_{n=2}^L ((k_{(n)})^2)^n + m^2]^\gamma}, \quad (10)$$

first perform the change of variables $r_{(n)}^2 = k_{1(n)}^2 + \dots + k_{m_n(n)}^2$. Second, change the variables to $z = r_{(n)}^2$ and then to $z' = z^2$. Collecting together these set of steps, we finally obtain

$$i_n = \frac{1}{2n\Gamma(\gamma)} S_{m_n} \Gamma(\frac{m_n}{2n}) \Gamma(\gamma - \frac{m_n}{2n}) (m^2)^{-\gamma + \frac{m_n}{2n}}. \quad (11)$$

Replacing this identity back into the expression for I , we can solve successfully all the integrals in higher powers of momentum along each competition subspace. Using $\epsilon_L = 4 + \sum_{n=2}^L \frac{(n-1)}{n} m_n - d$, we find

$$I = \frac{1}{2} S_{(d-\sum_{n=2}^L m_n)} \Gamma(d - \sum_{n=2}^L m_n) \Gamma(2 - (d - \sum_{n=2}^L m_n)) \left(\prod_{n=2}^L \left[\frac{S_{m_n} \Gamma(\frac{m_n}{2n})}{2n} \right] \right) \\ \times \left(t + \frac{u^* M^2}{2} \right)^{-\frac{\epsilon}{2}} \left(1 + \frac{\epsilon}{2} + O(\epsilon^2) \right), \quad (12)$$

which can be further simplified. In fact, developing the argument of the Γ -functions, using the identity $\Gamma(a + bx) = \Gamma(a)(1 + bx\psi(a))$ and recalling to absorb the angular factor already mentioned above, it is not difficult to show that the integral has the following singular structure

$$I = \frac{1}{\epsilon_L} \left(1 + \epsilon_L \left[h_{m_L} - \frac{1}{2} \left(1 + \ln \left(t + \frac{u^* M^2}{2} \right) \right) \right] \right). \quad (13)$$

We have to take another derivative of H_R with respect to M , which will produce the inverse susceptibility

$$\chi^{-1} = \frac{\partial H_R}{\partial M} = t + \frac{u^* M^2}{2} + \frac{u^*}{4} \left(t + \frac{3u^* M^2}{2} \right) \left[1 + \ln \left(t + \frac{u^* M^2}{2} \right) \right] + \frac{u^{*2} M^2}{4}. \quad (14)$$

For $T > T_L$ we substitute $M = 0$ and the coupling constant at the fixed point value $u^* = \frac{2\epsilon_L}{3} + O(\epsilon_L^2)$ into last equation, which produces the result

$$\chi(T > T_L) = \left(1 - \frac{\epsilon_L}{6} \right) t^{-(1 + \frac{\epsilon_L}{6})}. \quad (15)$$

Note that $\gamma_L = 1 + \frac{\epsilon_L}{6}$ and $\chi(T > T_L)$ above is consistent with scaling in the neighborhood of the critical point.

When $T < T_L$, we have to use the value of M at the coexistence curve which is defined by the condition $H_R = 0$, namely

$$M^2 = \frac{-6t}{u^*} + 3t[1 + \ln(-2t)]. \quad (16)$$

Replacing this value at expression (14) and neglecting $O(u^{*2} \sim \epsilon_L^2)$, we can demonstrate that the inverse susceptibility below the Lifshitz temperature has the form

$$\chi^{-1} = (-2t) \left(1 + u^* \left(1 + \ln[(-2t) + \frac{3u^* t}{2} (1 + \ln(-2t))] \right) \right) + \frac{3u^* t}{2} \ln(-2t). \quad (17)$$

Expanding the “logarithm of the logarithm” in the above expression using the expansion $\ln(1 + x) = x + O(x^2)$, employing the fixed point $u^* = \frac{2\epsilon_L}{3} + O(\epsilon_L^2)$ and neglecting $O(\epsilon_L^2)$, we obtain

$$\chi = (-t)^{-(1 + \frac{\epsilon_L}{6})} \frac{1}{2} \left[1 - \frac{\epsilon_L}{6} (4 + \ln 2) \right]. \quad (18)$$

Consequently, the susceptibility amplitude ratio is given by

$$\frac{C_+}{C_-} = 2 \left[1 + \frac{\epsilon_L}{6} (3 + \ln 2) \right] = 2^{\gamma_L - 1} \frac{\gamma_L}{\beta_L}, \quad (19)$$

where $\beta_L = \frac{1}{2} - \frac{\epsilon_L}{6}$.

This expression is exact at one-loop level, its functional form in ϵ_L for several universal classes is the same, but the latter encodes *distinct* universalities since $\epsilon_L = \epsilon_L(d, m_2, \dots, m_L)$.

III. AMPLITUDE RATIO FOR GENERIC ISOTROPIC COMPETING SYSTEMS

There are some minor modifications in the isotropic behaviors, but the trend to obtain the amplitude ratio follows the same script as in the anisotropic case. As there is only one subspace, say along $d = m_n$ space directions coupling n neighbors via alternate competing interactions, the bare density Lagrangian is given by

$$L = \delta_{0n} \frac{1}{2} |\nabla_{m_n} \phi_0|^2 + \sum_{n'=2}^{n-1} \frac{1}{2} \tau_{nn'} |\nabla_{m_n}^{n'} \phi_0|^2 + \frac{\sigma_n}{2} |\nabla_{m_n}^n \phi_0|^2 + \frac{1}{2} t_0 \phi_0^2 + \frac{1}{4!} \lambda_0 \phi_0^4. \quad (20)$$

As before, the isotropic critical region is defined by $\delta_{0n} = \tau_{nn'} = 0$ with $T \neq T_L$. There is only one renormalization group direction characterized by the ξ_n correlation length. We perform a dimensional redefinition in the momentum just as done in the discussion of the anisotropic behavior. The expansion parameter is now $\epsilon_n = 4n - m_n$. The renormalized free energy at one loop can be written in the form

$$F(t, M) = \frac{1}{2} t M^2 + \frac{1}{4!} g^* M^4 + \frac{1}{4} (t^2 + g^* t M^2 + \frac{1}{4} (g^* M^2)^2) I_{SP} + \frac{1}{2} \int d^{m_n} k \left[\ln \left(1 + \frac{t + \frac{1}{2} g^* M^2}{k^{2n}} \right) - \frac{g^* M^2}{2k^{2n}} \right], \quad (21)$$

and the nomenclature is almost the same as in the anisotropic case, except that now the integral I_{SP} given by

$$I_{SP} = \left[\int \frac{d^{m_n} k}{[(k + K')^{2n}] k^{2n}} \right], \quad (22)$$

is computed at the symmetry point $K'^{2n} = \kappa^{2n} = 1$. Performing a derivative with respect to M , we obtain

$$H_R = \frac{\partial F}{\partial M} = tM + \frac{1}{6} u^* M^3 + \frac{u^* M}{2} \left(t + \frac{u^* M^2}{2} \right) [I_{SP} - I], \quad (23)$$

with

$$I = \int \frac{d^{m_n} k}{k^{2n} (k^{2n} + t + \frac{u^* M^2}{2})}. \quad (24)$$

Let us compute this last integral by employing a Feynman parameter, which gives essentially Eq.(7) in the absence of the quadratic term. We then discover that the resulting

integral has the same pattern as Eq.(11) and can be solved along the same changes of variables in a identical manipulation which led to Eq.(12). The geometric angular factor which appears here is just the area of the m_n -dimensional unity sphere S_{m_n} and shall be absorbed in a redefinition of the coupling constant as before. Carrying out this procedure, we get to

$$I = \frac{1}{\epsilon_n} \left[1 - \frac{\epsilon_n}{2n} \ln \left(t + \frac{u^* M^2}{2} \right) \right]. \quad (25)$$

Now, we can calculate the integral I_{SP} either using the orthogonal approximation or exactly. Although we could determine the integral exactly, we would like to know the deviation between the two results. The reason is simple: in the computation of the exponents, only the anomalous dimension of the field had a significant difference: the first term is positive or negative depending the value of n in the exact computation, but it is always positive in the orthogonal approximation. Nevertheless, a numerical analysis proved that the maximal error for increasing space dimension and (arbitrally) fixed $\epsilon_n = 1$ occurred in the specific heat critical exponent for $n = 2$ (3.9%), increased for $n = 3$ for the same exponent (4.1%), but decreased for increasing n ($n = 4, 2.2\%$; $n = 5, 3.4\%$, ...) [26].

We have now, for the first time, the opportunity to test the effectiveness of the orthogonal approximation in amplitudes, which in our opinion is worthy analyzing in view of the facts already known from the deviations of critical exponents.

Next, let us perform the computation of the amplitude ratio using either the orthogonal approximation or the exact computation of the integral I_{SP} . As we shall see in a moment, the amplitudes above and below the Lifshitz temperature change.

A. Amplitude ratio using the orthogonal approximation

According to Ref. [26], the integral computed at the symmetry point utilizing the orthogonal approximation was shown to result in the expression $I_{SP} = \frac{1}{\epsilon_n} (1 + \frac{\epsilon_n}{2n})$. Hence,

$$I_{SP} - I = \frac{1}{2n} \left[1 + \ln \left(t + \frac{u^* M^2}{2} \right) \right], \quad (26)$$

which implies

$$H_R = \frac{\partial F}{\partial M} = tM + \frac{1}{6} u^* M^3 + \frac{u^* M}{4n} \left(t + \frac{u^* M^2}{2} \right) \left[1 + \ln \left(t + \frac{u^* M^2}{2} \right) \right]. \quad (27)$$

Taking another derivative with respect to M , we find

$$\chi^{-1} = t + \frac{u^* M^2}{2} + \frac{u^*}{4n} \left(t + \frac{3u^* M^2}{2} \right) \left[1 + \ln \left(t + \frac{u^* M^2}{2} \right) \right] + \frac{u^{*2} M^2}{4n}. \quad (28)$$

Now, $M = 0$ in last equation is the situation corresponding to $T > T_L (t > 0)$, or in other words

$$\chi^{-1}(T > T_L) = t + \frac{u^* t}{4n} [1 + \ln t]. \quad (29)$$

Replacing the fixed point value $u^* = \frac{2\epsilon_n}{3}$, the susceptibility above T_L reads

$$\chi(T > T_L) = t^{-\gamma_n} \left(1 - \frac{\epsilon_n}{6n}\right), \quad (30)$$

where $\gamma_n = 1 + \frac{\epsilon_n}{6n}$ is the isotropic susceptibility exponent. Below T_L , we determine the value of M in the coexistence curve defined by $H_R = 0$, which yields

$$M^2 = \frac{(-6t)}{u^*} + \frac{3t}{n} [1 + \ln(-2t)]. \quad (31)$$

Replacing this value in the expression of χ^{-1} , it leads to

$$\chi^{-1}(T < T_L) = (-2t) \left[1 + \frac{u^*}{4n} \ln(-2t) + \frac{u^*}{n}\right], \quad (32)$$

which at the fixed point $u^* = \frac{2}{3\epsilon_n}$ implies that we can write the susceptibility in the form

$$\chi(T < T_L) = (-t)^{\gamma_n} \frac{1}{2} \left[1 - \frac{\epsilon_n}{6n} (4 + \ln 2)\right]. \quad (33)$$

The susceptibility amplitude ratio which results from the above expressions is written as

$$\frac{C_+}{C_-} = 2 \left[1 + \frac{\epsilon_n}{2n} + \frac{\epsilon_n}{6n} \ln 2\right] = 2^{\gamma_n - 1} \frac{\gamma_n}{\beta_n}, \quad (34)$$

where $\beta_n = \frac{1}{2} - \frac{\epsilon_n}{3n}$ is the magnetization exponent.

B. Exact amplitude ratio

The main advantage of the isotropic case is that the Feynman integrals can be computed exactly. Thus, we can obtain the susceptibility amplitude ratio without the necessity of using approximations. The I_{SP} integral was already computed in Ref. [26] at the symmetry point and was shown to be given by the expression $I_{SP} = \frac{1}{\epsilon_n} [1 + D(n)\epsilon_n]$, where $D(n) = \frac{1}{2}[\psi(2n) + \psi(1)] - \psi(n)$. First, using Eq. (25) we find

$$I_{SP} - I = D(n) + \frac{1}{2n} \ln \left(t + \frac{u^* M^2}{2}\right), \quad (35)$$

which turns out to result in the following magnetic field

$$H_R = \frac{\partial F}{\partial M} = tM + \frac{1}{6} u^* M^3 + \frac{u^* M}{4n} \left(t + \frac{u^* M^2}{2}\right) \left[2nD(n) + \ln \left(t + \frac{u^* M^2}{2}\right)\right]. \quad (36)$$

It is easy to show that the inverse susceptibility which follows can be written as

$$\chi^{-1} = t + \frac{u^* M^2}{2} + \frac{u^*}{4n} \left(t + \frac{3u^* M^2}{2} \right) [2nD(n) + \ln(t + \frac{u^* M^2}{2})] + \frac{u^{*2} M^2}{4n}. \quad (37)$$

Hereafter we are going to use the coupling constant at the fixed point, i.e., $u^* = \frac{2\epsilon_n}{3}$. Set $M = 0$ for $T > T_L$ in the above equation in order to find the susceptibility in the following form

$$\chi(T > T_L) = t^{-\gamma_n} \left(1 - \frac{D(n)\epsilon_n}{3} \right). \quad (38)$$

For $T < T_L$ the value of M in the coexistence curve is given by

$$M^2 = \frac{(-6t)}{u^*} + \frac{3t}{n} [2nD(n) + \ln(-2t)]. \quad (39)$$

Substitution of this value into the expression for χ^{-1} results in the following value for the susceptibility below T_L

$$\chi(T < T_L) = (-2t)^{-\gamma_n} \left[1 - \frac{\epsilon_n}{2n} \left(1 + \frac{2nD(n)}{3} \right) \right]. \quad (40)$$

Using Eqs. (38) and (40), we finally obtain

$$\frac{C_+}{C_-} = 2 \left[1 + \frac{\epsilon_n}{2n} + \frac{\epsilon_n}{6n} \ln 2 \right] = 2^{\gamma_n-1} \frac{\gamma_n}{\beta_n}, \quad (41)$$

which is the same value obtained using the orthogonal approximation Eq. (34). As happened to the critical exponents at one-loop order, the orthogonal approximation and the exact computations of the susceptibility amplitude ratio in the isotropic case yield the same value. However, we do expect deviations in both calculations at two-loop order and beyond.

IV. CONCLUSION

The obtained anisotropic amplitude ratios maintain the same functional form as its counterpart in the Ising-like universality class, with the parameter $\epsilon_L = 4 + \sum_{n=2}^L \frac{(n-1)}{n} m_n - d$ replacing the ordinary perturbation parameter of noncompeting systems $\epsilon = 4 - d$. This leads to the property of universality class reduction. This property already appeared in the computation of exponents and is expected to be valid at all loop orders.

In fact, if we turn off all the competing interactions, i. e., by setting $m_3 = \dots = m_L = 0$, keeping just alternate couplings among second neighbors and identify $m_2 = m$, we obtain the result for the anisotropic m -axial universality class. Note that the ratio has the same

functional form for all values of $m \neq d$ and reproduces the uniaxial case $m = 1$ studied earlier [37]. For instance, three-dimensional systems have perturbative parameters $\epsilon_L = 1 + \frac{m}{2}$ which change at distinct values of m . Consequently, they produce different values for the amplitude ratios which is consistent with the universality hypothesis previously stated. Besides, if go on and switch off the competing interactions among second neighbors ($m = 0$) we obtain the result of the Ising-like universality class.

The isotropic amplitude ratios, on the other hand, possesses its own version of universality class reduction. Different values of the number of neighbors coupled via alternate couplings (n) are responsible by the variation of the susceptibility ratio. The case $n = 2$ corresponds to the isotropic m -axial ($d = m$ close to 8) universality class. Our result is the first computation of isotropic amplitude ratios, perhaps because isotropic systems were found only in lower dimensional systems ($m = d = 3$) so far, as proposed in the critical behavior of homopolymer-diblock copolymer mixtures [31, 34, 36, 38], which makes the perturbative parameter rather large ($\epsilon_2 = 5$). Let us try to extract meaningful results from our results for those three-dimensional systems.

Although the value of the amplitude above the critical temperature is not universal, let us compare the two values using the approximate and exact results for the isotropic case $n = 2$. Using $\epsilon_2 = 5$ in Eq.(30) we find $C_+ = 0.583$ using the orthogonal approximation, whereas the exact computation from Eq.(38) using $D(2) = -\frac{1}{12}$ yields $C_+ = 1.13$, and the deviation is huge. Nevertheless, comparing with the Table III from Ref. [36] both values are allowed. In fact for diblock polymer composition $\Phi_{DB} = 0.072$ at temperature $69.2 \pm 0.1^\circ C$ the measured amplitude is given by $C_+ = 0.6 \pm 0.04$ which is compatible with the value obtained via the orthogonal approximation. On the other hand, for a slightly change of composition, namely $\Phi_{DB} = 0.073$ measured at temperature $69.5 \pm 0.2^\circ C$ the amplitude value is $C_+ = 0.94 \pm 0.07$, which is also consistent with the exact amplitude. Notice that even though those authors confirmed the absence of the isotropic Lifshitz point, they considered the Lifshitz critical region *with the inclusion of the fluctuations* using SCFT (Ref. [34]) in their experimental fits of the susceptibility curves, which is quite a different method than the one proposed in the present work [39].

Therefore, this is the first solid indication that field theory renormalization group results including the contribution of fluctuations are consistent with experiments in those sort of polymers, in spite of the large value of the perturbative parameter. Though the deviations

between the amplitudes are significant and expected from their nonuniversal feature, the experimental results do not rule out the orthogonal approximation result. This is the first experimental ground to test the deviations in both calculations. But we can go on and compare the true universal susceptibility exponent obtained in the experiment with our previous two-loop calculation from Ref.[26]. The orthogonal approximation for $(N = 1, d = m = 3)$ yields $\gamma_2 = 1.90$, whereas the exact exponent is $\gamma_2 = 1.50$. The latter is consistent with the experimental value $\gamma_2 = 1.55 \pm 0.15$ obtained from the isotherm at 69.5°C with concentration of diblock copolymer at $\Phi_{DB} = 0.071$. It is amazing that the experiment carried out on the homopolymer-diblock copolymer considered by those authors can really be described using the isotropic Lifshitz universality class and its critical region, in spite of the large value of the perturbative parameter for three-dimensional systems. Perhaps the use of other field-theoretic isotropic results already (and to be) developed in other experiments to be performed might be successful in refining our knowledge of the Lifshitz critical region for these systems.

Another aspect is the theoretical possibility of occurrence of up to 6th character Lifshitz points in AB/BC mixtures of diblock copolymers [35]. If this system can be fabricated in the laboratory, our work represents a prevision of results for the susceptibility with increasing values of the perturbation parameter for three-dimensional systems, in analogy to what was studied in Ref. [36] using small angle neutron scattering. This is rather encouraging an evidence to pursue further universal aspects of this kind of critical behaviors using this field theoretical language, for instance, amplitude ratios above and below the critical temperature. This could shed new light in devising experimental applications to our model in order to measure those effects in a real physical system, with isotropic or (less obvious) anisotropic critical behaviors.

The universality class reduction in the isotropic case is even more evident than its anisotropic counterpart. As a matter of fact, $n = 1$ corresponds the system without competition and belongs to the Ising-like universality class. Therefore, systems without competition can be understood as special cases either from the anisotropic cases ($m_n = 0, n = 2, \dots, L$) or from the isotropic cases $n = 1$, a property already discovered in the computation of the critical exponents.

Since the isotropic ratio can be computed approximately and exactly as well, we calculated the ratio using both procedures for the sake of comparison of the deviations for individual

amplitudes and how this deviation could be understood at least in the case $n = 2$. The amplitudes themselves are different in both cases, but the ratio is equal. This property also takes place in the determination of critical indices using perturbation theory, but the result is valid only at one-loop level. We expect that both ratios will have deviations at two-loop order.

The most interesting extension of the method proposed here is the study of the specific heat amplitude ratio for generic competing systems, generalizing the discussion carried out for the anisotropic m -axial critical behavior[37]. It would be nice to tackle the computation of other universal amplitude ratios either at one-loop level or to extend the method to compute amplitude ratios at two-loop order [41] for generic competing systems.

Last but not least, we hope that the present investigation can be significant to motivate experimental techniques in order to determine the susceptibility amplitude ratio in magnetic systems such as MnP , $Mn_{0.9}Co_{0.1}P$ [42, 43], etc.. In addition the new phase encountered in MnP [44] and $Mn_{0.9}Co_{0.1}P$ [43] might be related to new effects of competition as described in the present work.

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